Displacement functions for anisotropic materials with gradients in elastic properties and their applications

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This talk will present development of three-dimensional analytical solutions to some problems of solid mechanics for functionally graded materials (FGMs). FGMs are advanced composite materials with properties that vary from one surface of the material to the other as a result of an intentionally introduced gradient in the composition of the material. Gradual variation of material properties is known to improve structural integrity and performance while preserving thermal, and/or structural benefits of constituent materials. The concept of graded material, initially developed for super heat resistant materials to be used in spacecraft or nuclear fusion reactors, is now actively explored in many other engineering applications, such as functional materials for energy conversion, dental and orthopedic implants, sensors and thermogenerators, to name but a few. Analysis of the mechanical behavior of materials with gradients in elastic properties is associated with considerable mathematical difficulties, but if the material is also anisotropic, i.e. having properties that vary with direction, the analysis becomes even more challenging. It is well-known that the three-dimensional elasticity problem of equilibrium of a deformable solid requires in general the solution of a set of fifteen coupled partial differential equations. However it may be simplified significantly by introducing appropriately chosen displacement functions. While for homogenous solids many displacement functions exist, it is not the case for inhomogeneous solids. The talk will show on how stresses and displacements in an inhomogeneous transversely isotropic solid, with constant Poisson's ratios and the same functional form of dependence of Young's and shear moduli on the co-ordinate normal to the plane of isotropy, can be represented in terms of two displacement functions [1, 2]. Then, it will be shown how this representation can be used to derive three-dimensional elasticity solutions for transversely isotropic functionally graded plates [3] and sandwich panels with graded core [4]. The combined effects of anisotropy and inhomogeneity on stress and displacement fields in plates and sandwich panels with gradients in elastic properties will be examined and discussed.

[1] M. Kashtalyan, J.J. Rushchitsky. Revisiting displacement functions in three-dimensional elasticity of inhomogeneous media. International Journal of Solids and Structures 46 (2009) 3463-3470.

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[3] B. Woodward, M. Kashtalyan. Three-dimensional elasticity solution for bending of transversely isotropic functionally graded plates. European Journal of Mechanics A/Solids 30 (2011), 705-718.

[4] B.Woodward, M. Kashtalyan. A piecewise exponential model for three-dimensional analysis of sandwich panels with arbitrarily graded core. International Journal of Solids and Structures (2015, under review)